Matrix models for the black hole information paradox

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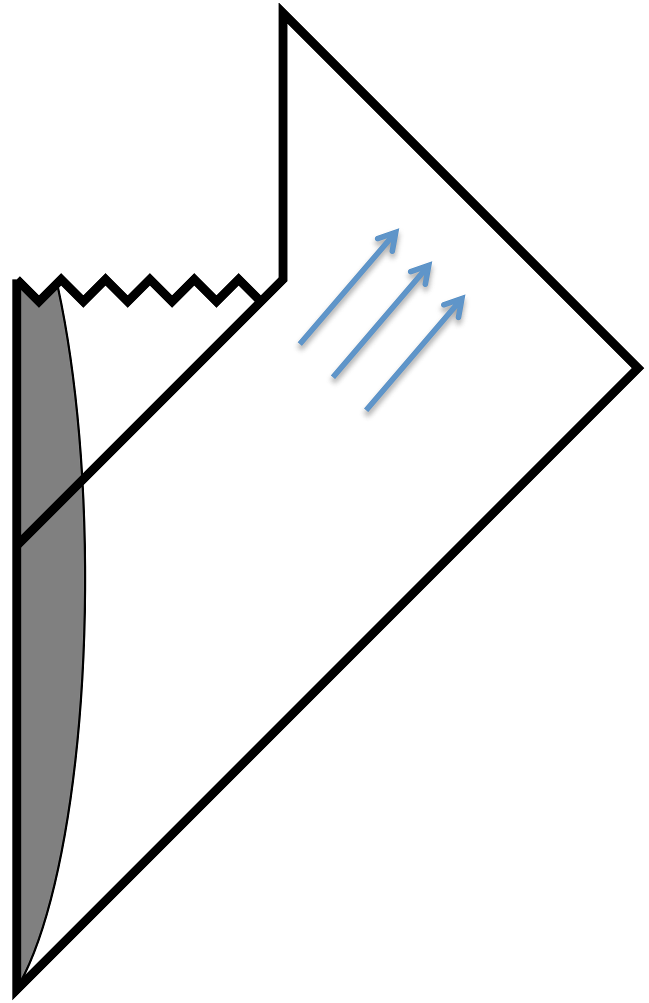
Joint work with N. Iizuka and J. Polchinski

- Black hole information paradox
  - Hawking's paradox for evaporating black holes
  - Maldacena's paradox for eternal black holes
- Matrix models for black holes
  - Exponential decay of a two-point function (review)
  - Perturbative 1/N corrections do not restore information

Information paradox for evaporating black holes

(Hawking)

1. Collapsing matter produces a black hole.
2. A black hole in Minkowski space evaporates by emitting radiation.
3. Semiclassically, radiation is thermal, and no information is stored there.



Possible outcomes of the paradox

- Radiation is in a pure state and there are phase correlations.

- Information is lost.

- A remnant with a huge number of states.

Possible outcomes of the paradox

- Radiation is in a pure state and there are phase correlations.

  ~~locality~~

- Information is lost.

  ~~principles of quantum mechanics~~

- A remnant with a huge number of states.

  infinite pair-creation of black holes

AdS/CFT

- For most of us, especially string theorists, AdS/CFT resolved the paradox: time evolution is unitary, and there have to be phase correlations.
- A small black hole in AdS evaporates, and the process is described, in principle, by a unitary evolution in gauge theory.

Information paradox for eternal black holes

(Maldacena)

- A large black hole in AdS does not evaporate, so there is no information paradox of Hawking.
- A related paradox: a two-point correlation function shows an exponential fall-off in AdS. Valid for large N and g^2N.
- In gauge theory at finite N, there must be recurrences.
- Questions: Which corrections in AdS restore recurrences?

- Festuccia and Liu argued that the exponential decay in the planar limit and recurrences at finite N persist to weak coupling g^2N.
- Though individual Feynman graphs do not have exponential decay, the radius of convergence in g^2N seems to go to zero at late times.
- Can we do better? $\mathcal{N}=4$ SYM is difficult.

→ Toy models

Goals:

1. Exponential decay (Iizuka and Polchinski)
2. 1/N corrections (Iizuka, TO and Polchinski)

Cubic model: exponential decay

(Iizuka and Polchinski)

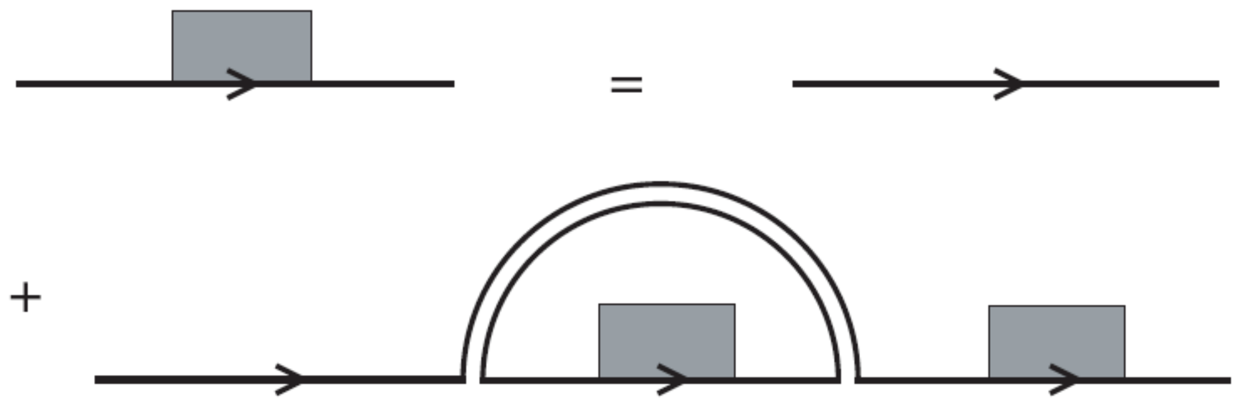
- The simplest possible model:
  - One adjoint field $X \propto A - A^\dagger$
  - One fundamental field $\phi \propto a - a^\dagger$
- The Hamiltonian is

$$mA^\dagger_{ij}A_{ji} + Ma^\dagger_i a_i + g a^\dagger_i (A + A^\dagger)_{ij} a_j$$

- Consider the two-point function at finite T

$$e^{iM(t-t')} \left\langle \mathrm{T}\, a_i(t) a^\dagger_j(t') \right\rangle_T \equiv \delta_{ij} G(T, t-t')$$

- The Schwinger-Dyson equation is given graphically as



- This leads to the recursion relation for $\tilde{G}(\omega)$:

$$\tilde{G}(T,\omega-m)-\frac{4}{\nu_T^2}\frac{1}{\tilde{G}(T,\omega)}+e^{-m/T}\tilde{G}(T,\omega+m)=\frac{4i\omega}{\nu_T^2}$$

with $\nu_T^2=\dfrac{2g^2N}{m(1-e^{-m/T})}$.

- At zero temperature, the recursion relation simplifies, and the exact solution can be found:

$$\tilde{G}(\omega) = \frac{2i}{\nu} \frac{J_{-\omega/m}(\nu/m)}{J_{-1-\omega/m}(\nu/m)} \ , \quad \nu^2 = 2g^2 N/m \ .$$

- At finite temperature, an analytic solution seems difficult to get. However, the recursion relation can be used to numerically obtain the solution.

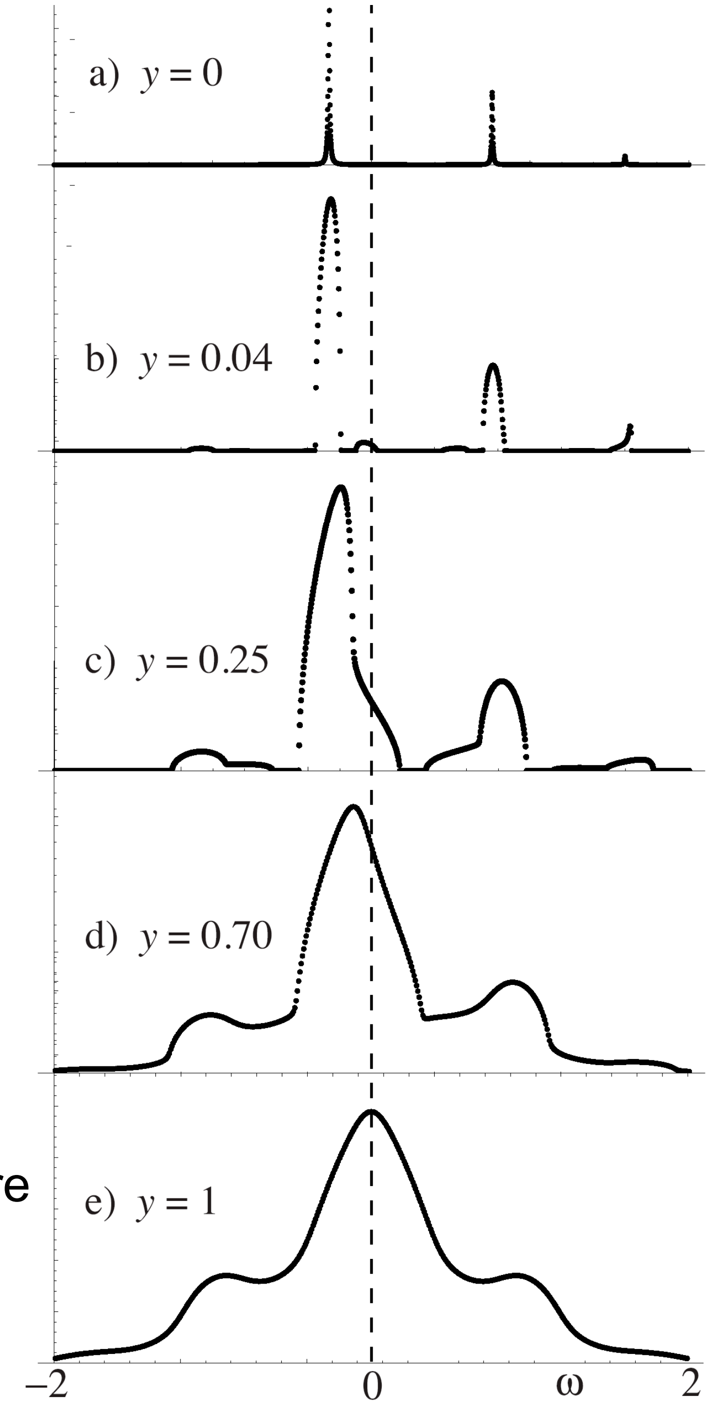
Zero temperature

Poles widen into cuts.
Cuts then merge.

$\mathrm{Re}\,\widetilde{G}(T,\omega)$ is shown.

$$y = e^{-m/T}$$

Infinite temperature



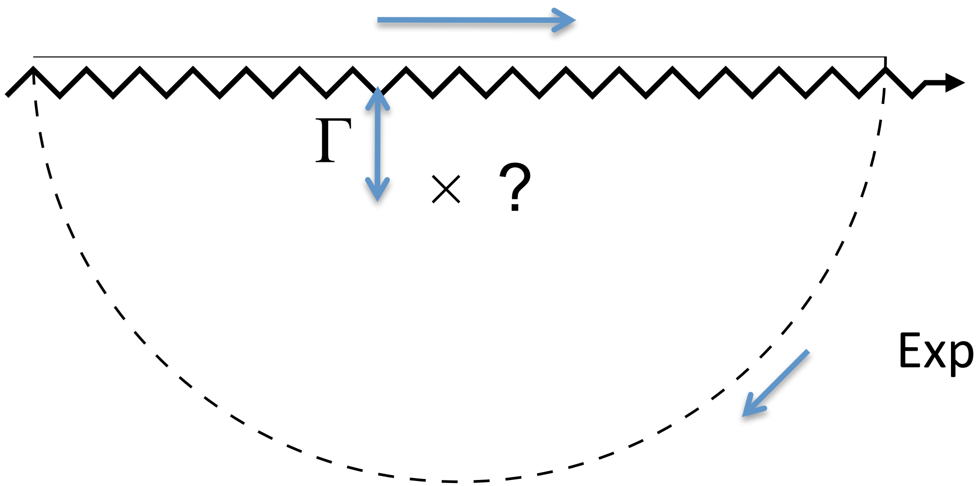
$$G(t) = \int d\omega e^{-i\omega t}\widetilde{G}(\omega)$$

- Branch cuts at low temperature:

$$G(t) \sim \int d\omega e^{-\omega t}\omega^{\alpha} \sim t^{-(\alpha+1)} \text{ as } t \to \infty$$

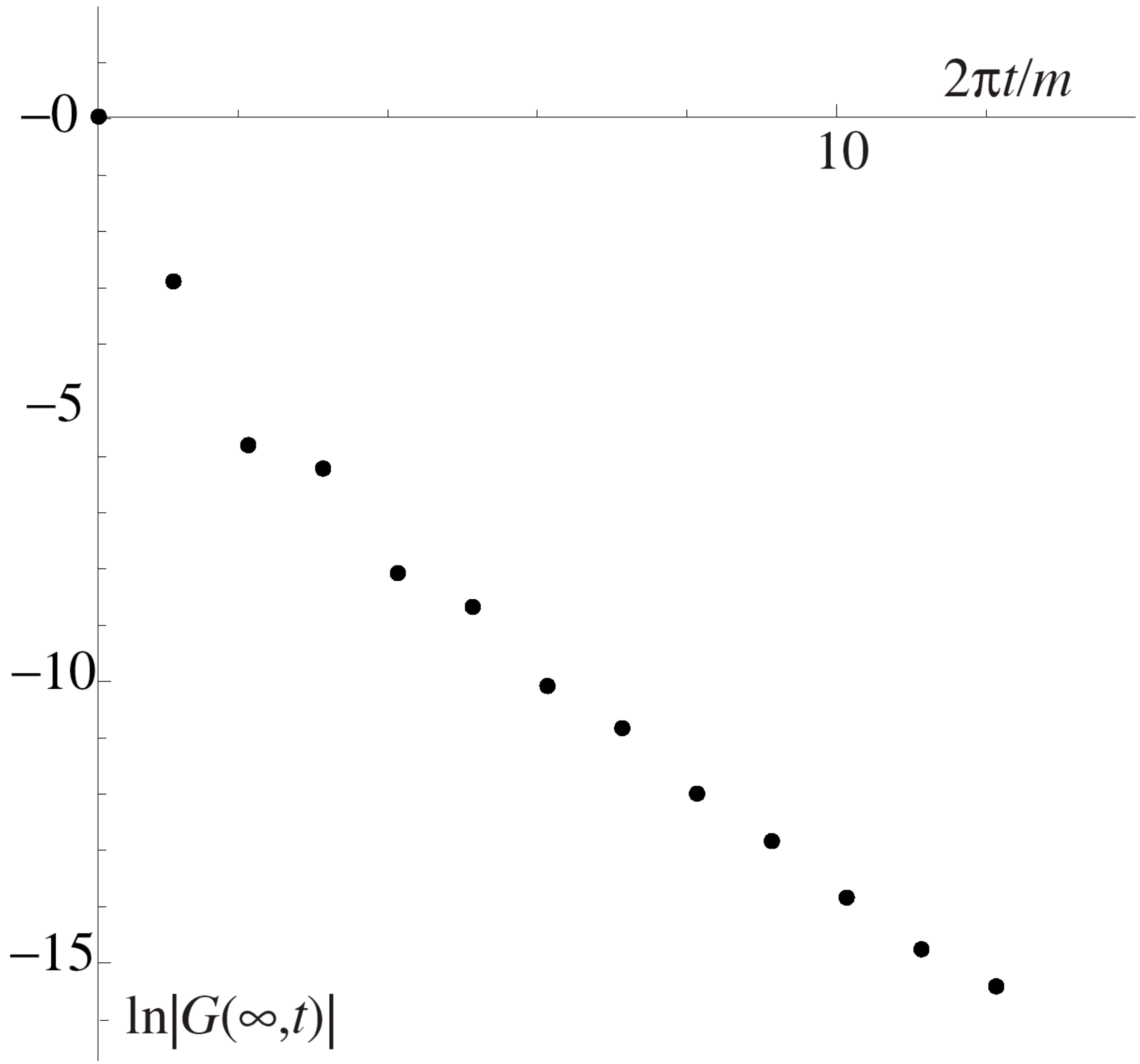
Power-law decay

- A single-cut along the real axis at high temperature



$$G(t) \sim e^{-\Gamma t}$$

Exponential decay



Exponential decay

Charge-charge model: perturbative 1/N corrections

(Iizuka, TO, and Polchinski)

$$H_{\rm int} = -h q_{li} Q_{il} \ , \quad Q_{il} = A^\dagger_{ij} A_{jl}$$

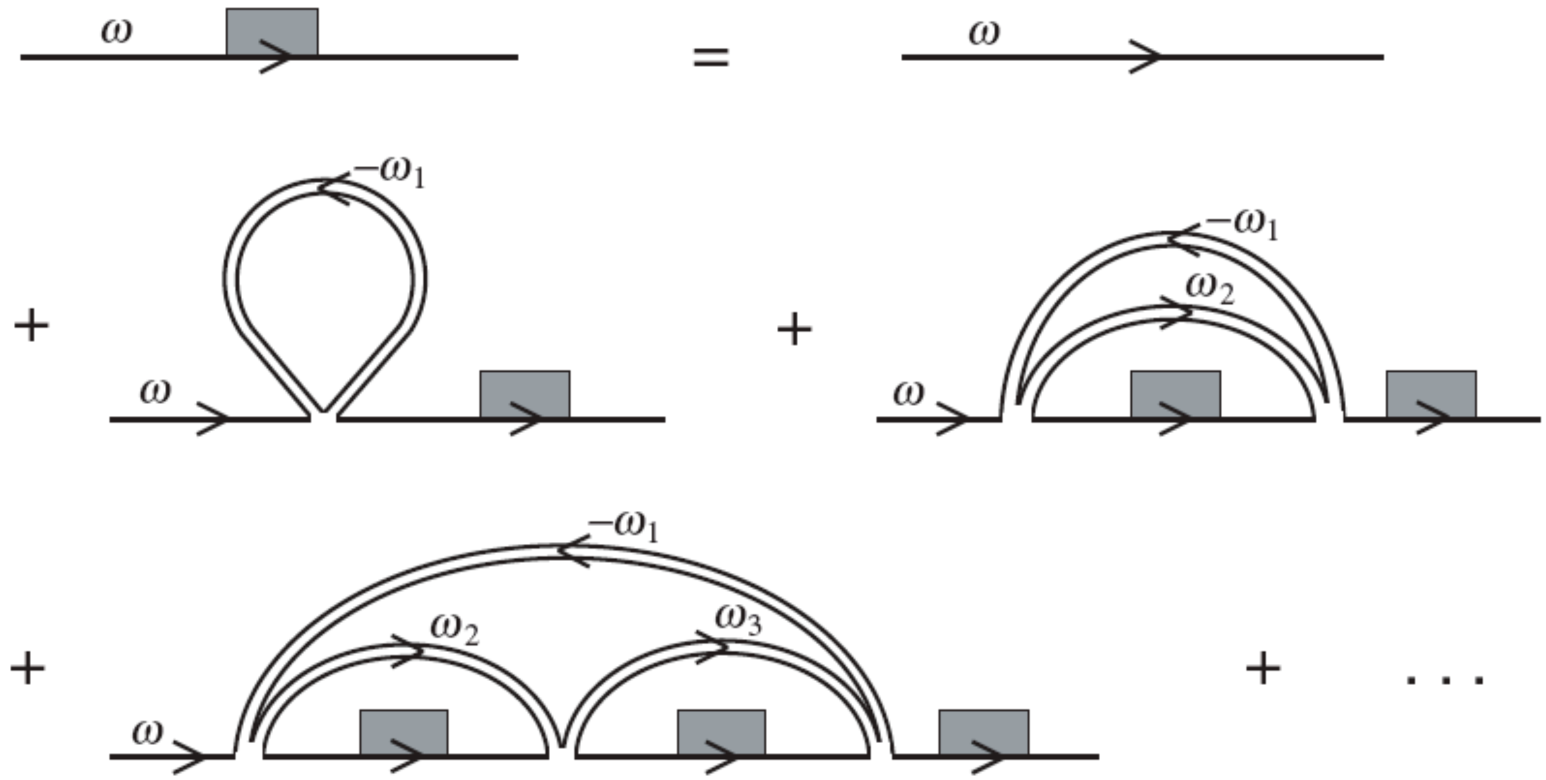
$$q_{li} = -a^\dagger_i a_l$$

Three different methods to analyze the model:

- Feynman diagrams and Schwinger-Dyson equations
- Loop equations
- Sum over Young tableaux

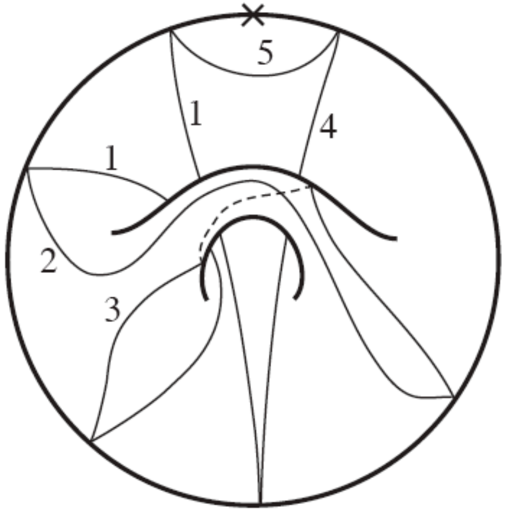
Feynman diagrams and the SD equations

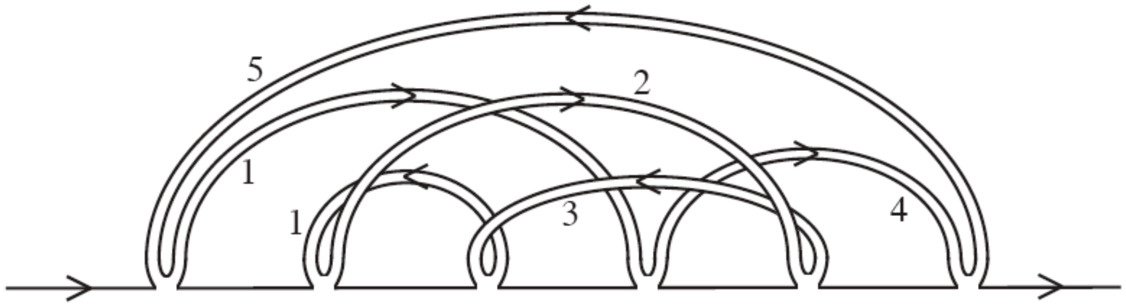
Genus zero



Feynman diagrams and the SD equations

Genus one





Feynman diagrams and the SD equations

Genus zero

$$\tilde{G}^{(0)}(T,\omega) = \frac{i(1-y)}{2\omega\lambda}\left(\lambda + \omega - \sqrt{(\omega-\omega_+)(\omega-\omega_-)}\right)$$

$$\omega_\pm = \lambda\frac{1+y\pm 2\sqrt{y}}{1-y}$$

Genus one

$$\tilde{G}^{(1)}(T,\omega) = \frac{iy^2x_0^3(1-x_0)^4(1-x_0[1-y])}{(1-2x_0+x_0^2[1-y])^4(\omega[1-x_0]^2-\lambda_y y)}$$

$$x_0 = -i\lambda_y\tilde{G}^{(0)}(\omega) \qquad \lambda_y = \frac{\lambda}{1-y} = \frac{hN}{1-y}$$

Loop equations

- For any operator $\mathcal{O}_{ji}$, the following equation holds.

$$\langle \mathcal{O}_{ji} A_{ij} \rangle = \frac{y}{1-y} \langle [A_{ij}, \mathcal{O}_{ji}] \rangle$$

- This relation can be used to compute

$$NG(t) = \theta(t) \left\langle \mathrm{Tr}\, e^{-ihQt} \right\rangle \ ,$$

$$N\tilde{G}(\omega) = \left\langle \mathrm{Tr}\, \frac{i}{\omega - hQ} \right\rangle \ .$$

- Genus zero and one contributions can be computed.

Sum over Young tableaux

- The charge-charge interaction can be written as a sum of quadratic Casimirs:

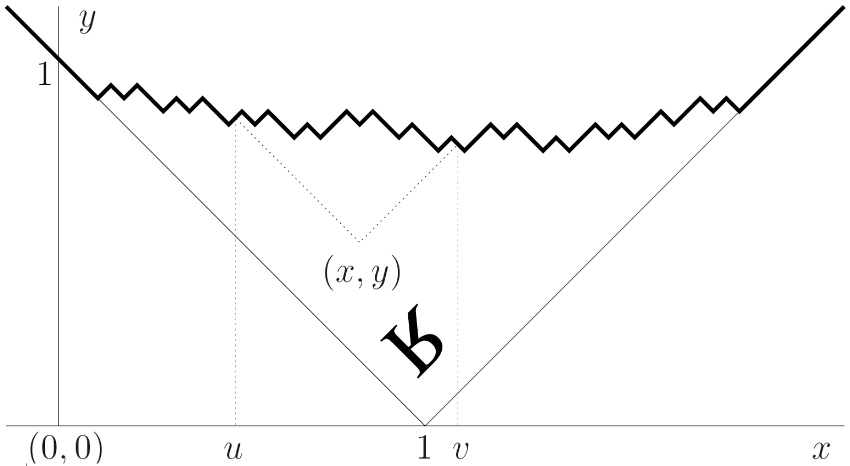
$$2q \cdot Q = (q+Q)^2 - q^2 - Q^2$$

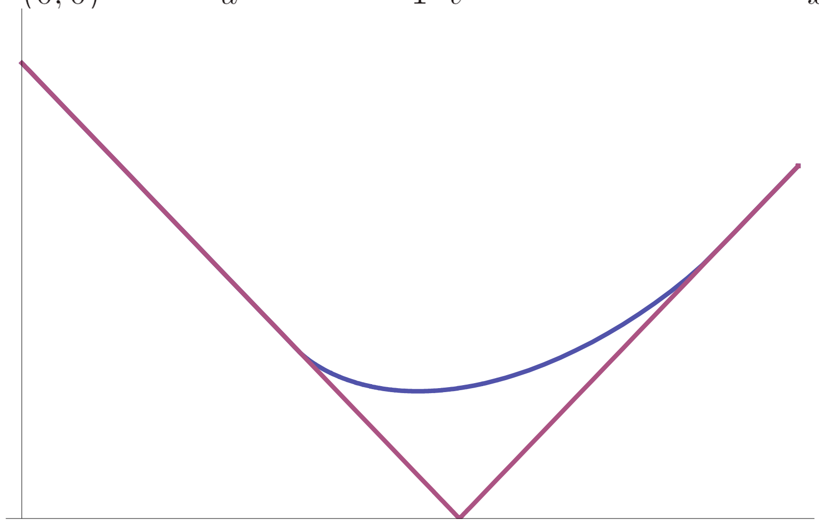
- The spectrum can be found by decomposing the Hilbert space into irreps of U(N).

$$-i\tilde{G}(\omega) = (1-y)^{N^2} \sum_R y^{|R|} (\dim R)^2 \; \Omega(\omega)$$

Sum over Young tableaux

In the large N limit, the sum becomes a functional integral over the shapes, and the genus zero amplitude is given by the typical tableau.





What restores information?

- Maldacena and Hawking conjectured that the sum over geometries (saddle points) restores information. This effect has size $\sim e^{-\mathcal{O}(N^2)}$.

- A known saddle point is the thermal AdS, which has the same boundary as the AdS black hole (Hawking & Page). The thermal AdS is expected to be realized as a saddle in the Polyakov loop integral (Aharony et al.).

- But in our models, we haven't included the Polyakov loop integral (= singlet constraint), so the second saddle is not the reason for information restoration.

Summary and conclusions

- Eternal black holes also exhibit an information paradox.
- Iizuka and Polchinski demonstrated exponential decay in a large N matrix model.
- Perturbative 1/N corrections do not resolve information loss, which requires non-perturbative effects.
- A second saddle does not restore information either.

(Beware of the artifacts of toy models)

Open problems and future directions

- Analytic understanding of the exponential decay.
- Matrix models for other problems. For example, look for a model with the largest Γ (fast scrambler).
- Exponential decay for open strings in the AdS black hole background.